USE OF A RING PROBE TO DETERMINE THERMAL CONDUCTIVITY COEFFICIENTS

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A method for using a constant-power ring probe to determine thermal conductivity coefficients is considered.

The transient constant-power cylindrical (linear) probe method is often used to determine thermophysical characteristics of liquids, gases, solids, and dispersed materials [1, 2]. The thermal conductivity coefficient λ of the material being studied is determined from the rate at which the cylindrical probe, used as both a heater and a temperature-sensitive element, heats up. At probe heating times longer than the characteristic time, which is dependent on heater radius, the probe heating rate, defined as the derivative of probe temperature with respect to the logarithm of time, will be constant. The thermal conductivity coefficient is then calculated from the expression

$$\lambda = \frac{Q}{4\pi\gamma}\text{, where } \gamma = d\Theta_{c,h}/d\ln t. \tag{1}$$

The measurement time is chosen within the limits $t_{min} < t < t_{max}$. The quantity t_{min} is determined by the thermal contact between probe and medium, while t_{max} is determined by the length of the probe or thickness of the specimen studied.

To perform such measurements, especially for determination of thermophysical characteristics of dispersed mediums such as soils, it is convenient to use a constant-power probe of annular form [3]. Such a probe configuration allows performing local measurements of thermal characteristics and has a number of technical and construction advantages as compared to the cylindrical form.

To justify the validity of use of such a probe we will consider the problem of the temperature field of a constant-power heat source in the form of a circle of radius R located within a homogeneous and isotropic medium with thermal diffusivity a and thermal conductivity λ. We choose the coordinate system such that the ring axis coincides with the plane XOY.

Using cylindrical coordinates we may write the temperature of an arbitrary point within the medium with an instantaneous source in the form of a circle of radius R, acting in the plane z = 0 at time t = 0 in the form [4]

$$\Theta_r(r,\ z,\ t) = \frac{QR}{4\lambda t\sqrt{\pi a t}}\exp\left[-\frac{r^2+R^2+z^2}{4at}\right]I_0\left(\frac{rR}{2at}\right). \tag{2}$$

If the ring probe is used as both heater and temperature sensitive element, we will be concerned with the change in temperature of the medium at the probe surface with heater radius r_h during heating. Denoting $r_h/R = \delta$, $u = 1/(2Fo)$, with consideration of the fact that $r = r_h + R$ and $z = 0$, we obtain

$$\Theta_r(u) = \frac{Q}{4\pi\lambda l}\sqrt{2\pi u}\exp\left[-u\left(1+\delta+\frac{\delta^2}{2}\right)\right]I_0[u(1+\delta)]. \tag{3}$$

Since $\delta \ll 1$ (in practice $\delta < 10^{-2}$) we obtain an approximate expression for $\Theta_r(u)$:

$$\Theta_r(u) = \frac{Q}{4\pi\lambda l}\sqrt{2\pi u}\exp(-u)I_0(u). \tag{4}$$

It can easily be shown that at large values of u (i.e., small t or large R) Eq. (3) tends to

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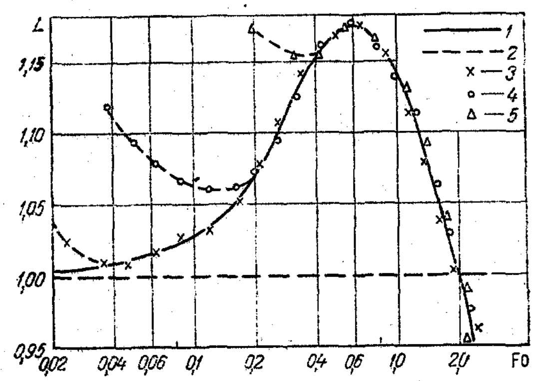

Fig. 1. Rate of medium temperature change at surface of constant-power probes: 1) ring probe (theory); 2) cylindrical probe (theory); experimental curves for ring probes of various radii: 3) R = 10 mm; 4) 8; 5) 4.

$$\Theta_{r}(t)=\frac{Q}{4\pi\lambda t}\exp\left[-\frac{r_{h}^{2}}{4at}\right], \tag{5}$$

i.e., it coincides with the temperature of the medium at the surface of an instantaneous linear source [4].

Equation (3) or (4) can be considered (at Q = 1) as a Green's function characterizing the ring probe located in a medium with known thermophysical characteristics. When a continuous ring source is used, an expression for temperature at its surface $\Theta_{r,h}(t)$ can be obtained in terms of a Duhamel integral, considering the initial temperatures of probe and medium identical and equal to zero. Then for $\Theta_{r,h}(t)$ we can write:

$$\Theta_{r,h}(t)=\int_{0}^{t}\varphi(t')\,h(t-t')\,dt',$$

where $\varphi(t')$ is a function describing the quantity of heat supplied to the probe over time; $h(t-t')$ is a Green's function, the form of which is defined by Eq. (4).

For a constant-power ring probe, i.e., at $\varphi(t') = Q = \text{const}$, introducing the integration variable $u' = R^2/2a(t-t')$, we obtain

$$\Theta_{r,h}=\int_{u}^{\infty}\frac{Q}{4\pi\lambda}\,\frac{\exp(-u')}{u'}\,\sqrt{2\pi u'}\,I_{0}(u')\,du'. \tag{6}$$

It is interesting to compare the heating rates of ring and cylindrical probes located in one and the same medium. For the quantity $d\Theta_{c,h}/du$ we can write

$$\frac{d\Theta_{c,h}}{du}=-\frac{Q}{4\pi\lambda}\,\frac{1}{u},$$

while for $d\Theta_{r,h}/du$, from Eq. (6) we have

$$\frac{d\Theta_{r,h}}{du}=-\frac{Q}{4\pi\lambda}\,\frac{\exp(-u)}{u}\,\sqrt{2\pi u}\,I_{0}(u).$$

Then from their ratio $L(u) = d\Theta_{r,h}/d\Theta_{c,h}$ we obtain

$$L(u)=\sqrt{2\pi u}\,\exp(-u)\,I_{0}(u). \tag{7}$$

The graph of L versus u or Fo can be displayed most conveniently in semilogarithmic coordinates (Fig. 1, curve 1). As has already been noted above, the heating rate γ of a constant-power cylindrical probe is a straight line parallel to the abscissa (curve 2).

Analysis reveals that the heating rate of a constant-power ring probe at sufficiently high u values (or small t) exceeds the cylindrical probe heating rate only insignificantly ($L \leqslant 1.05$ up to $Fo \leqslant 0.16$). After reaching a maximum value the ring probe heating rate begins to drop quite rapidly, tending to zero.

The condition for maximum L(u) can be written in the form

$$u = \frac{1}{2\left[1 - \frac{I_1(u)}{I_0(u)}\right]},$$

which corresponds to Fo ≈ 0.633, with L = 1.175.

To verify the results of this analysis, an experiment was formulated with ring probes of various diameters located in paraffin ($a = 0.150 \cdot 10^{-6}$ m²/sec, λ = 0.123 W/(m·°K)). Figure 1 shows normalized experimental heating rate curves $\Delta\Theta_{r,h}^{ex}$ for probes of various radius. The $\Delta\Theta_{r,h}^{ex}$ values are defined by the difference between probe temperatures at times t_2 and t_1 (t_2/t_1 = const) in a manner similar to that used in determining the heating rate of a cylindrical probe [5]. The quantity $\Delta\Theta_{r,h}^{ex}$ was normalized to its maximum value, which was taken equal to 1.175.

The satisfactory agreement between theoretical and experimental data should be noted, especially for Fo ⩾ 0.5. Deviation of the experimental curve from theory at small Fo values (the smaller the ring radius, the greater the deviation) can be explained by the finite time required for exit of the thermal wave into the medium (paraffin), the value of which depends on the ratios of the thermophysical characteristics of probe, insulating material, and medium under study, as well as the nonideal thermal contact between probe and medium. In our case, the ring was a tungsten wire ∿50 μm in diameter, surrounded by an Alundum sleeve with outer diameter of ∿1 mm for electrical insulation.

To determine the time t_{min} one can use estimates obtained in [5] for a cylindrical probe with insulating sleeve. Approximate calculations show that for a ring probe with Alundum insulation (λ = 2 W/(m·deg K)) to an accuracy of 5% t_{min} is equal to ∿20 sec, which is in agreement with experimental values.

Thus, the analysis performed shows that a constant-power ring probe can be used to determine thermal conductivity of a medium, using the measurement technique applied in the case of a cylindrical probe and based on Eq. (1). However, in contrast to the cylindrical probe, more severe restrictions must be placed on the maximum value of Fo, and thus on limiting measurement time t_{max}, as well as on dimensions of the ring probe. The maximum value of Fo is determined by the permissible deviation of the function L(u) from unity and is thus dependent on the tolerable uncertainty of the measurement. In practice, as was indicated above, if L(Fo) ⩽ 1.05, the value of Fo should not exceed 0.16.

NOTATION

λ, thermal conductivity; Q, power emitted into medium per unit probe length during heating; Θ_r, $\Theta_{r,h}$ $\Theta_{c,h}$ medium temperature at surface of instantaneous ring source and of ring and cylindrical constant-power heaters; t, time-measured from heater turn-on; t_{min}, t_{max}, minimum and maximum measurement times defining the measurement time interval over which the rate of change of medium temperature can be regarded as linear to a given accuracy; a, thermal diffusivity; R, radius of constant-power ring probe; r, z, coordinates of cylindrical coordinate system; r_h, heater radius; Fo = at/R^2, Fourier number; u = 1/(2Fo); I_0, I_1, modified Bessel functions with imaginary argument; L(u) = $d\Theta_{r,h}/d\Theta_{c,h}$; $\Delta\Theta_{r,h}^{ex}$ difference between probe temperatures at times t_2, t_1.

LITERATURE CITED

1. A. G. Shashkov et al., Methods for Determination of Thermal Conductivity and Thermal Diffusivity [in Russian], Énergiya, Moscow (1973).
2. A. F. Chudnovskii, Thermophysics of Soils [in Russian], Nauka, Moscow (1976).
3. F. Sh. Khodzhaev and A. F. Chudnovskii, "Soil-moisture-to-potential converter with annular probe," in: Devices and Equipment for Monitoring Plant Growth Media [in Russian], Agrofiz. Inst., Leningrad (1980), pp. 84-89.
4. H. S. Carslaw and J. C. Jaeger, Conduction of Heat in Solids, Oxford Univ. Press (1959).
5. A. F. Chudnovskii, "Measurement of semiconductor thermal conductivity by the transient probe method," Inzh.-Fiz. Zh., 7, No. 12, 23-31 (1964).